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SHORTER COMMUNICATION

INFLUENCE OF RADIAL CONVECTION ON BUBBLE GROWTH GOVERNED BY MASS TRANSFER

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INTRODUCTION

IN THE classical work of Epstein and Plesset [1] on the growth of spherical gas bubbles by mass diffusion, the influence of the convective transport due to the radial motion of the bubble wall was neglected. This effect was taken into account by Scriven [2] who formulated the coupled problem with simultaneous heat and mass transfer and gave some asymptotic solutions. In this paper the mass-transfer problem considered by Epstein and Plesset and Scriven is reexamined with the purpose of obtaining a simple criterion showing when the effect of the radial convection has to be taken into account.

FORMULATION OF THE PROBLEM

By assuming that the effect of liquid inertia and surface tension can be neglected, the equations governing the diffusion controlled bubble growth can be written [1, 2]:

$$\frac{\partial(r^2c)}{\partial t}+R^2\dot{R}\frac{\partial c}{\partial r} = D.\frac{\partial}{\partial r}\left(r^2\frac{\partial c}{\partial r}\right); \qquad r > R \tag{1}$$

$$c(r,0) = c_i \tag{2a}$$

$$c(R,t) = c_s \tag{2b}$$

$$\lim_{r\to\infty} c(r,t) = c_i \tag{2c}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\rho\frac{4\pi R^3}{3}\right) = 4\pi R^2 D\frac{\partial c}{\partial r}(R,t) \tag{3}$$

where c is the concentration, R the instantaneous bubble radius, D the diffusion coefficient, c_i the initial concentration, c_s the concentration at the bubble wall (assumed constant) and ρ the density of the gas in the bubble (assumed constant). Inherent in this formulation of the problem is the assumption that the density of the gas and the initial concentration are small compared to the density of the liquid.

ASYMPTOTIC SOLUTION

The mass-transfer problem as formulated admits a self-similar solution if only the asymptotic properties of the bubble growth are considered, [2]. The asymptotic solution of the bubble growth can be expressed

$$R = 2\beta\sqrt{Dt} \tag{4}$$

where β is an unknown parameter. By introducing the similarity variable

$$s = \frac{r}{2\sqrt{Dt}} \tag{5}$$

and making use of (4), equation (1) can be written

$$c''+2\left(s+\frac{1}{s}-\frac{\beta^3}{s^2}\right)c' = 0 \tag{6}$$

where prime denotes differentiation with respect to s. The solution reads

$$c(s) = c_i-2\rho\beta^3\exp(3\beta^2)\int_s^\infty\frac{1}{x^2}\cdot\exp\left(-x^2-\frac{2\beta^3}{x}\right)\mathrm{d}x \tag{7}$$

after applying the conditions (2a), (2c) and (3). The remaining condition (2b) determines the growth parameter β, i.e.

$$\phi(\beta) = \frac{c_i-c_s}{\rho} \tag{8}$$

where

$$\phi(\beta) = 2\beta^3.\exp(3\beta^2)\int_\beta^\infty\frac{1}{x^2}\cdot\exp\left(-x^2-\frac{2\beta^3}{x}\right)\mathrm{d}x. \tag{9}$$

DISCUSSION

The asymptotic bubble growth history when account is taken to the influence of the radial convection is determined by equation (4) and (8). If the radial convection is neglected we have according to Epstein and Plesset, [1]

$$\beta_0 = \left(\frac{c_i-c_s}{2\rho}\right)^{1/2} \tag{10}$$

when writing the equation for $R(t)$ on the form analogous to (4). The suffix "0" indicates that the radial convection is neglected.

In most applications, e.g. cavitation β is usually small, and thus $\phi(\beta)$ can be expanded in a series

$$\phi(\beta) = \beta^2-\pi^{1/2}\beta^3+O(\beta^4). \tag{11}$$

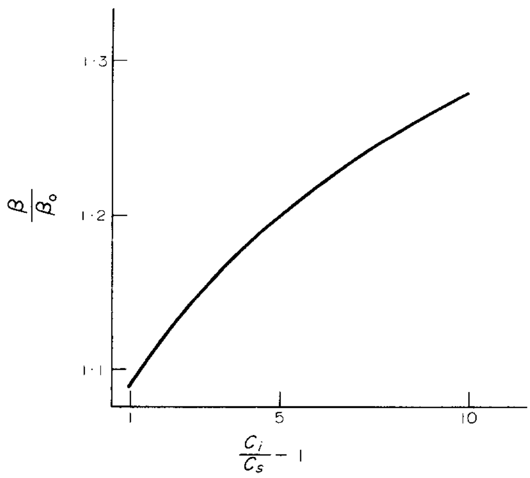

FIG. 1. Influence of radial convection; air–water (22°C), $c_s/\rho = 0{\cdot}02$.

The solution to (8) takes the form

$$\beta = \left(\frac{c_i - c_s}{2\rho}\right)^{1/2} + \frac{\pi^{1/2}}{4} \cdot \frac{c_i - c_s}{\rho} + O\left(\left[\frac{c_i - c_s}{\rho}\right]^{3/2}\right). \quad (12)$$

The influence of the radial convection is clear from a comparison between β and β_0, i.e.

$$\frac{\beta}{\beta_0} = 1 + \frac{\pi^{1/2}}{2}\left(\frac{c_i - c_s}{2\rho}\right)^{3/2}. \quad (13)$$

Figure 1 displays the ratio β/β_0 vs. $(c_i/c_s) - 1$ for air bubbles in water. As is seen the influence of the convection is of importance only in highly supersaturated solutions.

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